

**Exercise 26**

Find the limit or show that it does not exist.

$$\lim_{x \rightarrow \infty} \frac{x + 3x^2}{4x - 1}$$

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**Solution**

Multiply the numerator and denominator by the reciprocal of the highest power of  $x$  in the denominator.

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{x + 3x^2}{4x - 1} &= \lim_{x \rightarrow \infty} \frac{x + 3x^2}{4x - 1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \\ &= \lim_{x \rightarrow \infty} \frac{(x + 3x^2)\frac{1}{x}}{(4x - 1)\frac{1}{x}} \\ &= \lim_{x \rightarrow \infty} \frac{1 + 3x}{4 - \frac{1}{x}} \\ &= \frac{\lim_{x \rightarrow \infty} (1 + 3x)}{\lim_{x \rightarrow \infty} \left(4 - \frac{1}{x}\right)} \\ &= \frac{\lim_{x \rightarrow \infty} 1 + \lim_{x \rightarrow \infty} 3x}{\lim_{x \rightarrow \infty} 4 - \lim_{x \rightarrow \infty} \frac{1}{x}} \\ &= \frac{1 + \infty}{4 - 0} \\ &= \infty \end{aligned}$$